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ABSTRACT

The first section of this conference report attempts to set a context for the research and development recommendations made by the conferees and to reflect certain guidelines and cautions that participants felt were important. General background and some working assumptions, the present state of mathematics education as a context for conference recommendations, and some failed dichotomies, priorities for curriculum development, and tentative guidelines for research and curriculum development are covered in Section I. In Section II, twenty-two recommendations made by conference participants are recorded. First, the recommendations are listed under six categories: development of an information base, curriculum development for the immediate future, curriculum development for the long-range future, research and evaluation, teacher education, and dissemination. Then, for each of the recommendations, the conference discussion concerning that recommendation is reported and examples of suitable activities are given. Finally, the report concludes with summaries of presentations of resource persons. (DT)

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Report of the Conference on Needed Research and Development on Hand-Held Calculators in School Mathematics

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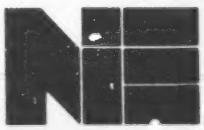

REPORT OF THE CONFERENCE ON NEEDED RESEARCH AND DEVELOPMENT ON HAND-HELD CALCULATORS IN SCHOOL MATHEMATICS

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PREFACE

The availability of inexpensive, hand-held electronic calculators as a result of recent technological advances, and the prospect of even more advanced equipment in the near future, have been of concern to mathematics educators. As a step toward clarifying the issues involved, the National Institute of Education and the National Science Foundation joined in inviting selected specialists to a conference held in Arlington, Va., from June 28 through 30, 1976.

The letter of invitation asked the participants to "produce a . . . planning document that will provide a well-defined framework for future research and development efforts . . . that NSF and NIE may use if they wish as a guide to program planning." However, the participants hope that this document will be useful to a much wider audience.

Thirteen persons attended the entire conference, and several others participated for shorter periods. Participants are listed on page ix. In addition, eight resource persons made presentations and were available briefly for consultation. The resource persons included representatives from the calculator and textbook publishing industries, and from a citizens' group opposed to calculator usage in schools. Other resource persons brought a wide range of experience in various aspects of research. (The names and titles of the resource persons, together with very brief summaries of their remarks, are included in a separate section starting on page 53.)

Aside from the resource persons' presentations, which were made to the entire group, most of the time at the conference was spent in small writing groups. These groups focused on specific issues related to research, curriculum development, and the gathering and dissemination of information. There was a considerable amount of shifting from group to group, so that each participant contributed to more than one.

The report that follows attempts to reflect the feelings and conclusions of the conferees. Wherever possible, the actual wording

supplied by participants is used. The report begins with an introductory section to set a context for the research and development recommendations made by the conferees, and also to reflect certain guidelines and cautions that the participants felt were important. Following that is a section that tries to set down in concise and structured form the research and development options that this group believed important in considering the potential offered by cheap and automatic calculation and computing power.

ACKNOWLEDGMENTS

We are indebted to the firm of Pacific Consultants and, in particular, to Judith Cox, for making the necessary arrangements before, during, and after the conference.

We are also grateful to Dean Bandes, who wrote an initial version of parts of this report, and to Max Bell and Marilyn Suydam, who were the principal authors of the final version.

Thanks are also given to the resource persons, all of whom added significantly to the breadth of the discussions at the conference. Finally, we thank the participants, who contributed so much to this report.

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**Chairperson.

I. INTRODUCTION

General Background and Some Working Assumptions

Small electronic calculators have been on the market in significant numbers only since about 1973, yet their impact on the lives of many people is already considerable. As an article in *Scientific American* put it, "For millions of people, everyday arithmetic will never be the same (McWhorter 1976)." Several authors have assessed the impact of calculators on the world external to schools (e.g., Suydam 1976). Many have predicted that by 1980 something like 80 million calculators may be in use in the United States, and that after 1980 perhaps 20 million calculators will be bought each year, many for replacement, trading up, duplicate ownership, and so forth. To put that in context, it may be noted that in 1974 there were about 70 million homes in the United States wired for electricity, nearly all of them with TV sets and refrigerators; there were 132 million telephones (95 million of them in homes); and there were 90 million automobiles. (There were also about 60 million students in schools and colleges.) Any instrument that seems destined within a few years to approximate the availability of refrigerators, TV sets, and automobiles is surely a force to be reckoned with.

But the prospect of widespread influence in the world *outside* of schools does not necessarily force schools to reckon with a given device. For example, whatever one might argue about indirect effects, television sets, refrigerators, telephones, or automobiles are directly present in the day to day curriculum of only a few classrooms. Yet participants in this conference arrived at this working conclusion very early:

> *The increasing availability of hand-held calculators in the student's world, at home and at school, forces educators to take a hard look to see what course the schools should take.*

One thing in support of this conclusion is that order of magnitude changes in technology have commonly been observed to cause fundamental changes in society. Calculators already represent at least an order of magnitude improvement in the speed with which many ordinary arithmetic calculations can be done, and at least two orders of magnitude decrease in cost, size, and weight over mechanical calculators with similar or lesser capabilities. But even more important is this fact:

> *These small, portable, and inexpensive machines have the potential for replacing the paper and pencil calculations that have been the major (and often the sole) component of elementary school arithmetic.*

This potential for replacing a large part of an existing curriculum makes this matter both urgent and controversial. It has also generated enough unease on the part of some influential parts of the public to lead to proposals to "ban the calculator" in schools. Similar if somewhat less fearful admonitions from within mathematics education itself have cautioned us to "use in early grades with care, if at all (Suydam 1976, p. 39)."[1]

But the matter does not end with the possible effects of four-function arithmetic machines that are already so available and so cheap.[2] Background articles and industry consultants convinced essentially all conference participants of even greater consequences:

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[1] *No consensus was reached at this conference on such a suggestion. Instead, participants agreed that whatever the short-term responses of schools might be, essentially all such matters should be subjected to the best possible development, trial, and research with youngsters in schools, with, of course, suitable safeguards against possible harm.*

[2] *The matter of price of calculators came up repeatedly in conference discussions. Beyond the order of magnitude effects already cited, one conference participant made an observation that may indicate why this is so. He remarked that in the school world, unit cost as such is everything--not such things as cost effectiveness or cost per pupil, but unit cost pure and simple. For example, many teachers can, if they insist, get "classroom sets" of something that costs $10 or less but often not even a single example of something that costs, say, $100 no matter what salutary effects the $100 item might have. Similarly, a building principal could perhaps buy $300 but not $3,000 items. Any such generalization must be taken with a grain of salt, of course, but it is worth thinking about, perhaps in connection with the sparse effects, so far at least, of such relatively expensive potential school "revolutions" as use of video in schools, or computer-assisted instruction.*

(Continued)

Microelectronic technology is changing at an astonishing pace. Today's four-function calculator will soon be replaced, at the same price, by one with many more functions. Today's scientific calculator will be replaced soon by comparably priced programmable calculators. Within a few years, models with memories comparable to today's medium-sized computers may replace merely "programmable" calculators, perhaps at only slightly higher prices than present scientific calculators. The latter, interfaced with larger computers, video, printing components, or other microelectronic devices may bring true computer capabilities to the home, school, or office at remarkably low prices.

Hence, there is little prospect for respite in considering the impact of calculators on school mathematics. By the time we have come to terms with pocket arithmetic machines we may need to cope with true pocket computers, and then perhaps with interactive visual displays (using TV sets) that at present are rare, even in large computing centers.

The advent of this kind of technology has been swift. The possible uses of such devices can affect what many people regard as the main purpose of much of elementary school mathematics instruction. This puts pressure on educators to respond to this change. Yet other supposed technological "revolutions" in the recent past have proved to be somewhat less than revolutionary. As a result, educators are cautious whenever new revolutions are announced. Conferees acknowledged the resulting conflicts:

Educators are faced with a dilemma. Their experience and instincts tell them to research, test, and proceed with caution. Yet calculator technology is progressing rapidly, and marketing pressures are great. The evolutionary pace traditionally associated with curriculum change is too slow to fit the present situation.

Whether too slow or not, that evolutionary process has already begun. There have been many small-scale school trials of calculators; at least one calculator manufacturer has published a program using calculators in the early school grades; various booklets on "how to use a

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The fact that the unit costs of simple calculators have just recently become quite low may thus remove some important barriers to widespread school use.

calculator" for this or that purpose have appeared; and some textbook publishers are marking "calculator suitable examples" in reprints of existing books, or revising teacher commentaries or making such other instant accommodations as they can in existing products. The conferees both recognized the value of evolutionary change and feared that it would be inadequate:

> *Whatever schools do or do not do, there will be increasing automation of calculation in society at large. Normal evolution of school practice could lead in 10 years or so to sound and fully acceptable school accommodation of the power presented by calculators. But this may not happen. Because schooling in mathematics (not merely in computation) is important, planned research and development will be necessary in the next decade to explore a variety of options that schools may use.*

The importance we attach to schooling in mathematics and not merely in computation was elaborated on by one of the working groups:

> *The present results of schooling in mathematics are far from satisfactory. Even if automated computation were not an inescapable prospect, new initiatives in the school mathematics curriculum would be in order. The advent of calculators makes such new initiatives imperative rather than merely desirable. Calculators may also create a climate of support for such initiatives in the face of the widely observed "back to basics" movement and more general misgivings about "innovation" in schooling.*

Part of the dilemma facing educators stems from the apparently contrasting characteristics of two major industries: the calculator and publishing industries. On the one hand, the microelectronics industry is rapidly developing new products and aggressively marketing them. Alternatively, published textbooks have traditionally dominated school curriculums and are, generally speaking, a conservative force in shaping the curriculum.

These characteristics present hazards that in some sense mirror each other. From the microelectronics industry side technologically naive school people could buy "hardware" without adequate supporting curriculum materials; the resulting unsatisfactory experience could inhibit truly productive use. On the publishing industry side, the time

and investment required to write, edit, and produce new or revised textbooks makes it likely that materials truly adapted to calculators will become available slowly. Furthermore, textbook publishers have made a huge investment in pre-high school textbooks that focus on teaching calculation. This investment is in some sense jeopardized by the statement that "for millions of people, everyday arithmetic will never be the same." While recognizing the contributions of the publishing industry, the conferees concluded that:

> *The education community, especially mathematics educators, must accept a leadership role in helping schools adjust to the existence of calculators. It must lead in delineating curriculum applications of hand-held calculators. It must not default, allowing manufacturers and publishers to make most of the crucial decisions.*

An effort to exert just such leadership became the dominant purpose of the conference. Some additional considerations that were discussed in that effort are outlined in the remainder of this introduction.

The Present State of Mathematics Education As a Context for Conference Recommendations

Many problems in mathematics education existed before calculators came on the scene. Some of those troublesome issues came up repeatedly during conference discussions. The following brief summary of these issues may aid in understanding the recommendations of the conference.

1. Every serious recent attempt to understand mathematics education and the means to its improvement has remarked on the shortage of so "simple" a thing as good information about what current practices and results really are. This is obviously a serious handicap and several of our recommendations aim to remedy it.
2. In much the same vein, even though there have been many research studies over the years, this body of research tends not to be known to many teachers and curriculum producers. One reason may be that related studies are too seldom reviewed

and made to converge on conclusions. Again, even when this happens, communication of such results is inadequate at best.[3]

As already noted, several past "revolutions" in mathematics education have come and largely gone, yet educators are little the wiser about what effect they had or could have had. Little insight has been gained about the *process* of change in education. Conference participants hope for better results as studies are done on use of calculators. Hence, several recommendations are aimed at information gathering and communication and at monitoring the course of events as schools succeed or fail in attempts to accommodate to calculators.

3. An old and persistent problem for mathematics education is the extent to which mathematics in the public mind *is* primarily paper and pencil calculation or, at most, strange manipulation of algebraic symbols. That it emphatically is not merely calculation or symbol pushing probably does not need argument for many readers of this report.[4] Conferees made it one of their recommendations to try to change this unfortunate public misperception. For one thing, if it is widely believed that mathematics is essentially calculation, the obvious corollary that will surely be proposed sometime is that mathematics can be purchased at the corner discount house for $8.95--with consequences for support of quality mathematics education that may be serious.

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[3] *Begle (NIE 1976, pp. 13-18) claimed that what research says about many of our perennial problems can be assessed and he offered several examples. It is, he said, simply a matter of someone going to work and doing the job. In many other parts of that report and also in the report from the National Advisory Committee on Mathematical Education (NACOME) problems of inadequate information and sparse conclusions from research were alluded to many times (Hill et al 1975).*

[4] *The 33 position papers in Conference on Basic Mathematical Skills and Learning (NIE 1976) took on "basic skills" in mathematics from a wide variety of viewpoints; essentially none identified basic skills (to say nothing of mathematics itself) merely with calculation skills. Many papers conterded that problem solving, estimation, functions, graphs, measurement, and simple statistics are truly "basic skills" for everyone, for reasons other than accommodation to calculator availability.*

4. Whatever mathematics itself is, few would dispute that in actual elementary school practice, *calculation* is the major curriculum goal (e.g., see Usiskin and Bell in Suydam 1976). This is unfortunate in any case, but widespread use of calculators outside of school raises obvious questions about such a major emphasis. There are parallel questions to be raised about the effects calculators may have on "skill and drill" emphases in secondary school subjects such as algebra. The questions are obvious and central to the whole debate about school use of calculators. Many recommendations here acknowledge these questions in one way or another. In general conference participants did not propose any particular point of view, but rather urged support of considerable diversity in innovative curriculum production and pedagogical invention with the calculator. Other recommendations insist that critical evaluation of cognitive and affective outcomes accompany development and trial of these new materials and pedagogies--an "obvious" but too often neglected matter.

5. Mathematics education has in the past failed to sort out what various psychological and learning theory models say about the learning of mathematics. Accommodating to calculators should raise those questions anew. For example, interpreters of Piaget have recommended "concrete experiences," particularly in early school mathematics. Will the concealed operations of calculators violate the need for concrete materials, or can calculators themselves become concrete learning aids? Alternatively, if one accepts behaviorist models, there appear to be obvious ways of using calculators or other programed microelectronic devices in their service. "Behavioral objectives" of the sort too often proposed for mathematics (e.g., "Given any three digit whole number, the student will find its square root to two decimal places") may be in trouble if the behavior consists of pushing the correct buttons.

Conferees discussed such matters and the recommendations here reflect a wish for considerable diversity in devising and testing calculator-conscious curriculum materials based on

explicitly stated theoretical positions. We hope that such explictness will lead to more clarity than in the past.

6. Essentially all past curriculum studies, as well as the ones we propose, assign importance to assessing "attitude" and other noncognitive outcomes, but there are too few really satisfactory mechanisms for doing so. Again, this is an "obvious" matter, and will not be discussed at length here. One of the recommendations in this report calls for a "status study" of what is available; another calls for development of improved means of studying such matters.
7. The limitations of most standardized tests have been a tolerated problem in mathematics education for some time. Because most such tests concentrate heavily on computational skills, the use of calculators makes these problems urgent. Clearly most standardized tests (and much classroom-based achievement testing) will be meaningless if children habitually use calculators in school and, as would seem natural, attack test problems with the same tools they use in learning. This report recommends a thorough and critical "status study" on the one hand and development of new testing initiatives on the other hand.
8. New curriculum and pedagogical initiatives succeed or fail at the classroom level. For too many promising innovations, the result is failure. No conferee was disposed to "blame" teachers for the inadequacy of elementary schooling in mathematics and the extent to which its emphases are out of kilter. Indeed, there was considerable sympathy for teachers who must bear burdens for which they are often inadequately prepared. But research and development efforts such as recommended here *must* involve teachers whenever possible. They must also provide teachers with as much information and guidance as possible on the promise and possible hazards of calculators. Teachers, especially those in elementary schools, must receive training and retraining to prepare them to cope with substantially changed emphases. The recommendations in this report reflect

these concerns. There is surely nothing new in them, but for once things should be done in the right way!

Some Failed Dichotomies

During the conference, one working group concentrated on research while another worked on curriculum development. Similarly, one group examined short-range development while another looked at the longer term. This split up conferees into manageable working groups and increased the diversity of ideas offered, but neither dichotomy could be maintained in the recommendations of the report. Research must go hand in hand with development, even for short-range projects. Developers should review relevant research in designing their curriculums, and researchers should investigate existing curriculum materials in choosing suitable contexts for their investigations.

For example, there is currently much interest in whether using calculators in the early grades will inhibit such things as learning the "basic facts" or computational algorithms. (Algorithms, of course, are the completely systematic methods for computing sums, products, and so on.) To investigate this, researchers must obviously deal with curriculum questions--that is, exactly how calculators and other materials are being used in a particular classroom. This sets the context for research efforts.

Likewise, a certain amount of curriculum development is frequently an integral part of the research effort, for any of several reasons. For example, the specific purposes of a given research effort may require curriculum materials that do not yet exist. As another example, a research effort might attempt to develop and test pedagogical inventions that overcome past difficulties or demonstrate the existence of new possibilities in learning and teaching. One instance of this is pedagogical inventions of the 1950's and 1960's that made functions much more accessible to young children than had been the case.

The dichotomy between short-term and long-term development suffered a similar fate; that fact requires some explanation. Participants recognized early that in responding to a new need such as accommodation to calculators, at least two forms of development are needed: development of small pieces, and development of whole courses or related sets of

courses. The former takes place wherever anyone sees a need. It tends to come in small bits; to have immediate use; to be more amenable to individual attack; and to be modest in cost. Many such adaptations will be required soon as educators learn the best ways to use calculators in schools. Recommendations for several "short-range" initiatives are contained in this report. But the boundary between long- and short-term developments is not clear. The experience gained through using "short-range" innovations in classroom practice can be very important to long-range, more radical development. Likewise, planning for the long run in any truly innovative way often requires feeling one's way and adapting as one goes along by creation and trial of a succession of smaller pieces. This lack of clear boundaries is reflected in the recommendations in this report.

Although short-term innovations are essential, some substantial part of any funding that might become available should go to development and research that try to anticipate long-term needs. Such projects should try to anticipate circumstances that are not immediately pressing but that will be important by the time development can be accomplished. We feel that such anticipation is possible and that some of the most thoughtful people in mathematics education should be attempting it.

Priorities for Curriculum Development

Calculators and their successors may eventually influence the entire school mathematics curriculum and perhaps school curriculums more generally.[5] This large question must be divided into smaller bits to permit priorities to be set. Participants discussed this matter, but reached no agreements. The following comments are notes on this discussion but do not constitute "recommendations."

With respect to carving up the problem, there are identifiable blocks to the mathematics curriculum as follows: grades K-3; 4-6; 7-8; 9-10 "terminal"; 9-10 "college preparatory"; 11-12 "specialist" or

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[5] *For example, calculators and eventually very cheap computers could enable science and social studies courses to incorporate the mathematical models and statistical methods that are frequently used in these disciplines.*

"college preparatory"; and 11-12 "nonspecialist," including many who go on to college.

Clearly, piece-by-piece development is appropriate for various parts of all these curriculum blocks. In particular, development with accompanying research is needed as soon as possible to determine whether there is any validity to widespread concerns that extensive use of calculators will somehow destroy children's reasoning abilities. (As one participant facetiously phrased the question, "Will use of calculators rot the mind?")

In addition, relatively expensive, full-scale, future-oriented development is needed for certain curriculum blocks, including certain separate mathematics courses at the secondary school level. Such development is surely needed for curricular areas where computation is either a major objective of instruction or a major prerequisite. Entire curriculum blocks are needed, for example, for both of the elementary school blocks; for the usual junior high school seventh- and eighth-grade courses; for general mathematics offered in grades 9 and 10; for consumer mathematics in the upper levels; and for a separate course in statistics that emphasizes "making sense of data" and uses a heavy data base and many practical problems.

As these new courses are developed, genuine attempts should be made to take advantage of ideas and products already in existence. Even though the calculator itself is a relatively recent device, many of the relevant mathematical notions have been incorporated into past development projects; such projects should be analyzed, and worthwhile pieces saved.

The time and money needed for such development will be substantial and will require financial support from either Federal or State sources. If both a need and a market for such courses can be demonstrated, publishers and calculator manufacturers may be enticed to help with the funding effort. But in many essential cases seed money and development funding must come from Federal or other noncommercial sources.

Use of calculators will require less revision of some current courses, such as high school algebra, geometry, and elementary functions and analysis. In all these courses, however, some parts need to be revised to include more applications that exploit the full potential of

calculators. Such revisions will be less costly in time and in money than development of whole new courses, and probably can be carried out by individuals, by publishers, by local school systems, or through efforts led by professional organizations. Over the long run, it may become evident that these courses themselves need major redevelopment as calculator use in schools matures; this may come even sooner as hand-held *computers* enter the scene.

Tentative Guidelines for Research and Curriculum Development

Though they agreed the widest possible range of options should be developed, conference participants found themselves stating probable starting points and "standards" for research and development proposals. Some were regarded as nearly absolute; for others, explicitly stated departures seem likely. Some tentative conclusions of this discussion are the following:

1. As something like an absolute standard, several of the working groups quoted Policy Recommendation 2 of the NACOME report (Hill, et al. 1975, p. 137):

> *Those who are concerned about the education of young people must reaffirm their commitment to full, comprehensive mathematics education for all youngsters regardless of race, national origin, or sex, and to the encouragement of each to pursue mathematics so as to make maximum use of his or her mathematical talents.*

In that connection one of the consultants who visited the conference observed that black and other minority group children in our poorer schools are sometimes made "guinea pigs" for so-called "innovative" research. Participants unequivocally recommend that research and development aimed at "special groups" formed on the basis of minority status *as such* should not be approved. Research with "special groups" formed across such rubrics as achievement, age and grade, and special mental or physical handicaps may be important, but assumptions about such things as race and sex as determinants of specialness are clearly to be avoided.

2. Some (but not all) of the usual expectations about the learning of counting, numeration, and number processing will probably remain as reasonable expectations even in the most calculator-dominated

curriculum. The following skills contribute in important ways to such things as "number sense," estimation skills, and understanding of our number system and its uses, and should continue to be expected of every student:

a. Ability to multiply and add one-digit numbers nearly automatically. A student who has not acquired these "reflexes" is nearly crippled, if only because rapid processing of small-number sums and products is so often a feature of rough and ready estimation.

b. Firm understanding of our standard numeration system, including reading and writing arbitrarily large or small whole numbers and decimals. As part of this, students should be familiar with notation involving positive and negative powers of 10 (though this is not presently expected), because it helps in estimation and because it is sometimes needed in using calculators.

c. Firm understanding of the various meanings of fraction notation (e.g., parts of a whole, ratios, division) and ability to exploit "equivalence" of fractions. Even though there will be more emphasis on decimal notation, fraction ideas remain essential.

d. Experience with algorithms. Algorithms in mathematics are an important concern in themselves. But the same sort of work and the same algorithms that are now used should not necessarily be retained. A student should probably have some systematic way to compute the sum or difference of any two numbers, and reasonable products and quotients. (An attempt to define "reasonable" is in appendix F of Suydam 1976.)

3. Research and development teams should normally include a mix of teachers, mathematics educators, mathematicians, evaluation specialists, and other specialists as appropriate. Such efforts should normally be closely linked to classrooms. Where only individuals or small groups are engaged in such work, it is hoped they will have access to other specialists and to schools.

4. Many people will regard *any* departure from present expectations as potentially harmful. But young people are resilient. Mistakes and temporary lapses need not be permanently harmful. Nevertheless, protection of research subjects is a real concern. Research or development projects likely to be seen as "high risk," in the sense that they depart widely from present practice, should be carried out initially only where unusual resources exist or can be provided.

5. In a similar vein, some research and development should be conducted under "most favorable" conditions, in order to establish the range and upper bounds of what can be taught. Here, "most favorable" refers to the extensiveness of the resources devoted to such efforts, not to any particular setting for the work. Indeed, some such work should be attempted in settings considered unfavorable in order to ascertain the full extent of what it is possible to teach.

6. Research and development proposals should be as explicit as possible in such areas as the following:

a. What provision is to be made for, or what deliberate departures from, such expectations as those outlined in item 2 above?

b. What additional content is the focus of the efforts in question?

c. What affective results are anticipated and how are they to be evaluated?

d. What pedagogical or theoretical positions constitute the underlying assumptions of the proposal?

e. In what ways, if at all, does the research or development effort propose to respond to stated needs of its subjects--the practical demands of the world they live in, for example?

f. What technology is to be used, or, if the effort focuses on likely future needs, what technology is assumed?

g. How will the final product and each stage of development be evaluated? Evaluation should often include truly critical monitoring and comment by knowledgeable teachers and other specialists.

h. To what extent have existing resources been surveyed?

7. With respect to evaluation, development efforts that approach the stage of widespread implementation should be subjected to well-designed randomized experiments to assess effects and effectiveness. This would be analagous to requirements of proof of effectiveness in pharmaceuticals or impact statements in environmentally sensitive development. Too many of today's school materials are used because of custom or good salesmanship rather than proved effectiveness. The same could happen in the case of calculators. Such experiments should not be limited to federally sponsored development. It is fair enough to encourage such research on commercially produced curriculum materials and honest reporting of results.

Conclusion

Without claiming to have given an exhaustive record of the discussions that led to the recommendations that follow, the conferees hope that this introduction gives some of the flavor of those discussions. It is further hoped that the report as a whole provides a good starting place for the many people who must now become involved in thoughtful consideration of what should happen in schools now that it seems certain that "For millions of people, everyday arithmetic will never be the same."

II. RECOMMENDATIONS

Recommendations in Brief

The recommendations are grouped into several broad areas of concern: development of information base, development of curriculum materials for the immediate future and for the long-range future, research and evaluation considerations, teacher education, and dissemination. They are numbered consecutively; the order in which they are listed does not suggest priorities.

A. Development of an Information Base

Recommendation 1. *Existing mathematics curriculum materials should be intensively and critically analyzed to determine whether they can be adapted for use with calculators.*

Recommendation 2. *Current instructional practices should be studied to determine what actually happens in school mathematics classrooms.*

Recommendation 3. *Existing research literature and other available information on student achievement and student attitudes in mathematics learning should be reviewed.*

Recommendation 4. *Actual uses of ordinary calculation in the everyday and working lives of people should be surveyed.*

Recommendation 5. *A long-term study of the calculator as an educational innovation should be undertaken soon as a way of providing insight into innovation in education.*

B. Curriculum Development for the Immediate Future

These recommendations obviously do not exhaust the possibilities; they merely attempt to identify some immediately pressing or important needs. Many things developed for the immediate future will also support development aimed at the long-range future.

Recommendation 6. *Materials should be developed to exploit the calculator as a teaching tool at every point in the curriculum to test a variety of ideas and possibilities pending emergence of calculator-integrated curriculums.*

Recommendation 7. *Research and development should be undertaken with respect to algorithmic processes in school mathematics, especially those in arithmetic and algebra.*

Recommendation 8. *Curriculum materials should be developed for kindergarten through twelfth grade to teach estimation, approximation, significant digits, order of magnitude calculations, and similar ideas.*

Recommendation 9. *Curriculum materials should be developed that teach problem-solving strategies more effectively and that build pupils' confidence in their ability to solve problems.*

Recommendation 10. *Curriculum materials should be developed and tested on new topics that are not now in the curriculum but that calculators may make feasible. The possibilities of adapting "advanced" topics to earlier levels should also be explored.*

C. Curriculum Development for the Long-Range Future

Recommendation 11. *Some full-scale alternatives to the K-6 elementary school mathematics program should be developed that use calculators wherever appropriate (but not exclusively) and that considerably broaden the range of mathematical ideas absorbed during the elementary school program.*

Recommendation 12. *Some new courses for secondary school students should be developed, including consumer-industrial and data-oriented statistics courses, as well as alternatives for junior high school work and for those high school students who do not elect standard "college preparatory" mathematics courses.*

D. Research and Evaluation

Research and evaluation must interact with curriculum development at every point. Besides that general interaction, the conference identified these special needs.

Recommendation 13. *Sampling and research methodology should be appropriate to the type and scale of development. Methods can range from clinical trials to full-scale randomized intervention experiments, and from research focused on special subsets of the school population (e.g., remedial, handicapped, gifted) to that focused on a more general population.*

Recommendation 14. *Current psychological, behavioral, and learning theory models should be surveyed to identify what they might contribute to the curriculum development process.*

Recommendation 15. *There should be development and research focused on new approaches to achievement testing. Testing policy should be analyzed and appropriately changed in response to these results.*

Recommendation 16. *New approaches to assessing attitudes and other non-cognitive outcomes of school mathematics instruction should be studied, and curriculums using calculators should be subject to such assessment.*

Recommendation 17. *Every effort should be made to assess any changes in the amount of time spent in mathematics instruction and the distribution of that time as calculators become an integral part of the school experience.*

E. Teacher Education

The conference intended the single recommendation that follows to suggest interaction of teacher education with the recommendations above at every possible point.

Recommendation 18. *Training and retraining of teachers to help them respond to calculators and calculator-influenced curriculum materials must be made equal in importance to the development of initiatives aimed at school-age children.*

F. Dissemination

Recommendation 19. *A central information collection and dissemination center should be established.*

Recommendation 20. *Ways should be explored to inform the public about how schools are adapting to calculators.*

Recommendation 21. *New means should be explored to rapidly communicate results of experiments with calculators and proposals for their use to the teaching profession, especially to elementary school teachers.*

Recommendation 22. *Means should be found to ensure access to critical evaluations of calculator-oriented materials proposed for school use, and to provide information and guidelines about calculators themselves.*

Recommendations with Discussion and Examples

This section takes each of the recommendations and elaborates them with a record of conference discussion about each and with examples of suitable activity offered by the conferees. These elaborations can obviously not be regarded as giving complete or definitive statements on each recommendation, nor can the amount of discussion for given recommendations be interpreted as indicating relative priorities. In the limited time available, some things were talked about more than others

and some important matters proved resistant to making specific suggestions.

The recommendations are grouped into certain broad areas of concern: development of an information base; curriculum research and development efforts for the immediate future and for the long-range future; other research and develpment concerns; teacher education; and information sharing. They are numbered consecutively; the order of listing does not suggest priorities.

A. Development of an Information Base

The recommendations in this category are in large part aimed at "state of the art" assessments of mathematics education. They differ from other recommendations dealing with dissemination. The recommendations in this section call for active, deliberate attempts to sort out what is the present situation in mathematics education and to keep track of how that changes over the next few years in response to calculators and other influences.

Such attempts will interact with the information sharing initiatives proposed by the final group of recommendations by making use of things collected by the Calculator Information Center and, of course, by contributing the findings of the various assessments that will be conducted. Also, the information base developed by the recommendations here will be a starting place and resource for many curriculum development and research efforts.

Recommendation 1. *Existing mathematics curriculum materials should be intensively and critically analyzed to determine whether they can be adapted for use with calculators.*

Discussion and Examples.--The goals or major objectives of the curriculums should be examined by groups including mathematics curriculum specialists, teachers, mathematicians, and psychologists. Information, including both data and points of view, should be collected for examination; in addition, thoughtful attention should be given to the appropriateness of goals, perhaps especially to the role of computational processes. Articulation across levels should be considered.

The purposes of these analyses are: (1) to provide a basis for deciding how to modify existing curriculums to incorporate use of the calculator; (2) to provide a base for developing and testing the effectiveness of new materials and sequences; and (3) to provide a base from which to develop hypotheses for research.

The task of analysis could be accomplished in a variety of ways. For example, much progress could be made in a 1-week working conference preceded by information collection. Alternatively, months-long studies, by one or several groups, could be commissioned either separately or in conjunction with curriculum development projects.

The analyses should identify and discuss such factors as:

1. With which topics or components of the curriculum could calculators be used?
2. With which of these components might use of calculators be especially feasible to facilitate learning?
3. With which of these components should calculators not be used?
4. Are curricular demands on students particularly extensive at some points? Usiskin and Bell, for example, suggest that there is a considerable escalation in symbolic complexity in grades five and six (Suydam 1975).
5. Do the materials provide clues to identify the hardest pedagogical problems? Can calculators help with them?
6. Which components of the curriculums are noncomputational?
7. How do the needs of society and of the subject interact to affect curriculum priorities? As an example, one could consider the move toward use of the metric system, treatment of decimals in current curriculums, and the decimal emphases indicated by calculators.
8. Do any present curriculum materials claim to respond to particular theoretical models? For example, do any materials claim to be based on what children are developmentally ready to learn?
9. What curricular prerequisites appear to be firm even though calculators are used?

10. What topics in the existing curriculums need increased attention?
11. What topics not in the existing curriculums might be incorporated?
12. Which of the modifications or types of restructuring seem most crucial?

Recommendation 2. *Current instructional practices should be studied to determine what actually happens in school mathematics classrooms.*

Discussion and Examples.--As noted in the NACOME report (Hill et al 1975), little specific information exists on current instructional practices. Therefore, surveys should be designed to answer such questions as:

1. What is being emphasized in classrooms?
   a. How much time do teachers at each level actually spend teaching mathematics?
   b. How much time do teachers spend teaching computation and other topics with which the calculator could be used? How much time is spent on other mathematical topics?
   c. How much time do students spend on each aspect of mathematics (i.e., in class, in independent work, on homework, on out-of-school activities, etc.)?
   d. How extensively are calculators now used in classrooms at various levels?
   e. What are currently successful uses of calculators and teaching procedures for using them?
   f. How are teachers coping effectively with having many different types of calculators in the same classroom?

   To obtain such information, written reports alone cannot be accepted as valid; observations are essential.
2. What is emphasized in instructional materials?
   a. How much is included in textbooks and other materials on computation (and other topics with which the calculator could be used)? How much is included on other mathematical topics?

b. What is the amount of emphasis placed on each of the above?

c. What is the quality and usefulness of print materials designed for use with calculators?

This involves a survey of curriculum guides, textbooks, and so on. Some of this information is already available, for example, from the Pennsylvania Retrieval of Information for Mathematics Education System (PRIMES).

3. What is emphasized on standardized tests?

a. What emphasis is placed on the various topics above?

b. What is the relationship of emphases on tests and emphases of teachers and instructional materials?

Recommendation 3. *Existing research literature and other available information on student achievement and student attitudes in mathematics learning should be reviewed.*

Discussion and Examples.--Although there have been thousands of studies related to mathematics education, little attention has been devoted to drawing conclusions that can serve to guide curriculum development efforts. While much has been learned from research, not all research nor all research reports have focused on issues of interest to many curriculum developers. Indeed, some issues are so big that finding answers takes much combined effort.

Additional emphasis must be placed both on coordinating research and on synthesizing the results of research. Begle (NIE 1976, pp. 13-18) is among those who feel that something definite could be said on quite a variety of issues by pulling together and abstracting findings from the research literature. This is true independent of any effects from calculators, of course, but the challenges and opportunities for change presented by calculators may make more urgent some attempt to sort out what is known from research on children's mathematical learning.

It would seem profitable to convene a small group of people to identify some questions that might have answers, to point out those places where reviews have already synthesized definite conclusions, and

to suggest what efforts, if any, would be most fruitful in this area. It is possible to profit more from existing research studies than many seem to believe we have.

Recommendation 4. *Actual uses of ordinary calculation in the everyday and working lives of people should be surveyed.*

Discussion and Examples.--Little research evidence has been collected on how and to what extent computational skills are used in everyday life since the studies conducted by Guy Wilson and his students between approximately 1918 and 1930. Opinions abound; data are needed to determine societal needs, both current and projected, for computational skills as well as for other mathematical topics. Surveys should be designed to answer such questions as:

1. How and when are computational skills used in everyday life by students from kindergarten through twelfth grade? By adults for vocational and nonvocational purposes?
2. How often are paper-and-pencil algorithms used?
3. How often are estimation and mental computation used?
4. What mathematical skills are necessary and desired by today's society? What is the degree of agreement among students, parents, teachers, and business representatives?
5. How and when do children and adults use calculators in everyday life?
6. What mathematics is needed by students and adults given widespread availability of calculators?

The following considerations should be kept in mind in responding to this recommendation:

1. Observations must be conducted; merely asking people how or whether they use mathematics is not sufficient.
2. Not only should number of uses be tabulated, but the specific uses of each skill, algorithm, and so on, should also be noted.
3. Another payoff from these surveys can be the collection of "real-life" problems and applications.

4. While surveys may concentrate on present uses, projections should be made of anticipated uses and shifts in emphases. These can be based on anticipated technological progress, for example.

Recommendation 5. *A long-term study of the calculator as an educational innovation should be undertaken soon as a way of providing insight into innovation in education.*

Discussion and Examples.--As noted earlier in this report, educational innovations all too frequently are ballyhooed for a while and then disappear with little trace. Little is learned about the process of change itself. Sometimes there are improvements on this, as for example with several after-the-fact studies of "the new math" curriculum reform movement of the 1960's, but even these have been retrospective.

It is easy to predict that calculators will cause quite a stir over the next 5 to 10 years. Anticipating that, researchers should begin as soon as possible to document the course of this event in education and to follow it through as a case study of how change does or does not occur in educational practice. Such recording needs to begin soon, before "recall" becomes the primary means of noting behaviors, and while "real-time" recording is still possible.

Among the sources of information are printed references (both articles and advertisements) from educational journals, popular magazines, news magazines, technical journals, newspapers, and project reports. Information must be analyzed and synthesized at periodic intervals with a view to graphing progress and determining trends.

One other important source of information would be the reactions of various individuals and groups. To assess those, analytic surveys of opinions related to use of calculators should be conducted among school professionals, persons in the publishing and calculator industries, other special groups to be identified, and the public at large. A beginning effort of this sort is in Suydam (1976), but that report merely summarizes reasons used by those for and against using calculators in schools. An in-depth analysis would be needed to determine the nature (i.e., composition, characteristics) of those individuals and groups

who oppose or favor various types of use of calculators in schools at various levels. Such information may be especially helpful in designing strategies for change.

As part of the overall effort, researchers should monitor how use of calculators affects student attitudes, skill retention, and other mathematical understandings as times goes on. While the National Assessment of Education Progress (NAEP) will provide some of this information for a random national sample, smaller-scale surveys need to be aimed at collecting more explicit information more frequently. State departments of education, working with local educational agencies, can help to provide such information by administering common tests, such as are involved in State assessments, with calculator use controlled and data examined particularly in terms of the impact of the calculator. Other researchers from various geographic regions or types of communities might coordinate efforts to develop (1) a framework for conducting such surveys, (2) agreements on test components and items, (3) common data analysis techniques, and (4) common forms of reporting. Such surveys address public concerns, and can allay fears and put concerns into focus. Potential effects of calculator use, both positive and negative, need to be investigated, and findings must be reported in readily understood terms.

Another area of investigation might be how student attitudes are affected by the way parents use calculators, and, in the same vein, how parents view *school* use of calculators as a result of how they themselves use calculators.

Clipping services could be used to collect printed information; agencies specializing in historical analysis could aid in interpretations. Polling agencies outside the educational community may be especially useful in designing and conducting opinion surveys. It may be possible to "piggyback" questions relevant to this recommendation on other regular surveys of opinion; for example, Gallup and Roper; the National Opinion Research Center periodic surveys; efforts of groups studying public reaction to automation and other technological changes; and so on. Modest funding soon of a small planning group might be a first step in getting this underway. It should be noted that NAEP will

attempt to get a preliminary data base with respect to calculator use and effects of that use in its 1977 assessment of mathematics.

B. Curriculum Development for the Immediate Future

The recommendations in this and the following section are directed toward resolving questions about the use of the calculator to assist students in developing mathematical ideas and skills in a variety of school situations.

In general, curriculum development efforts should be directed toward producing (1) calculator-related components that can be used with current curriculums and (2) curriculums in which the use of calculators is totally integrated. The focus of this section is on the first of these, capitalizing on existing knowledge, practices, and applications to formulate fruitful present uses of calculators. Some questions discussed in this section may be of concern for, say, an interim 5-year period, while new calculator-oriented curricular materials are being developed. However, many of these interim findings and materials will also make direct contributions to revision of curriculums for the long-range future.

As noted in the introduction, research and curriculum development are interwoven processes. Fruitful curriculum development depends on creative reflection on the question of what needs to be taught and how to teach it. Research provides information on psychological and mathematical factors that might help shape an instructional strategy. Research also provides information on the extent to which various materials and sequences that have been developed enhance or inhibit the learning of mathematics or affect attitudes toward mathematics.

The following recommendations obviously do not exhaust the possibilities for work directed at the immediate future. They merely attempt to identify some immediately pressing or important needs. Many things developed for the immediate future will also support development aimed at the long-range future.

Recommendation 6. *Materials should be developed to exploit the calculator as a teaching tool at every point in the curriculum to test*

a variety of ideas and possibilities pending emergence of calculator-integrated curriculums.

Discussion and Examples.--The analyses of existing practice proposed in recommendations 1 and 2 will presumably lead to the type of development suggested here. For instance, on the basis of recommendations from those analyses, research and development groups might tackle such immediately pressing questions as these in the primary grades (K-4):

1. Does using calculators in the primary grades interfere with or aid in the acquisition of counting skills, number concepts, and other mathematical ideas?
2. Does using calculators require close teacher supervision to be constructive at the primary level? Does unsupervised use of calculators facilitate learning?
3. What mathematical ideas will primary grade children learn independently without direct instruction?
4. What is the effect of using "growing keyboard" calculators compared with calculators having nonrestricted keyboards?
5. What does the availability of calculators allow primary grade children to learn that they would not learn without calculators? For example, do they improve in problem solving or in logical thinking?
6. What learning does the use of calculators prevent primary school children from acquiring? For example, do they learn the "basic facts"?

More generally, research and development studies need to be designed to provide evidence on such immediate questions as these with respect to the entire school range (K-12):

1. What mathematical topics can best be taught with the calculator? ("Best" might be defined in terms of efficiency, ease, speed, achievement, and other factors.) When should calculators not be used?
2. What are some long-range effects of calculator use on learning, recall, and use of "basic facts"? Of algorithms? Of other

mathematical ideas? How can calculators best be used to enhance mastery of essential topics?

3. What algorithms are most appropriate to use with calculators and in what ways can the calculator be used to lead to the acquisition of algorithms and to the learning of algorithmic processes? (See recommendation 7.)
4. If students use calculators that are more sophisticated than their current levels of instruction, will students investigate independently and bypass so-called prerequisite topics?
5. If teachers restrict students from using certain keys (e.g., percent, square root) until a developmental process has been followed, what will be the effects?

Certain priorities for development of supplementary materials were designated by one of the working groups. These priorities are for materials that do the following:

1. Aid in the acquisition of essential mathematical ideas and skills (for all students).
2. Aid in applying mathematics to consumer problems, career needs, and interdisciplinary concerns.
3. Aid in the acquisition of problem-solving strategies.
4. Aid in the learning of other important mathematical ideas, such as the following:
   a. Number patterns and the properties deduced from them. For instance, a student studying the commutative property will be able to verify that 1473 X 2981 = 2981 X 1473 as easily as that 3 X 5 = 5 X 3. Patterns can be expanded and completed; limit processes can be explored.
   b. Decimal numeration.
   c. Measurement.
   d. Graphing. This topic can be made more meaningful when students can see that every point on a graph corresponds to an (x,y) pair that they can calculate.
   e. Maps, involving ratio and proportion.
   f. Integers.
   g. Basic elements of statistics, including data collection and analysis.

h. Probability.

i. Variables.

j. Functions.

Recommendation 7. *Research and development should be undertaken with respect to algorithmic processes in school mathematics, especially those in arithmetic and algebra and perhaps in trigonometry.*

Discussion and Examples.--What are possible alternative algorithms? What criteria might be appropriate to determine which algorithms will be taught? Both in the past and at present, educators have been concerned with the speed with which an algorithm can be performed and the probability that students will get the correct answer. The use of calculators (as well as computers) enables a shift of focus to the algorithmic process and algorithmic learning. Thus the clarity of an algorithm's logic perhaps assumes more importance than its efficiency.

Another criterion is the adaptability of the algorithm to calculator evaluation. For instance, calculator polynomial evaluation is done most effectively in a nested-parenthesis form: $((ax + b)x + c)x + d$ rather than $ax^3 + bx^2 + cx + d$. That same arithmetic can be done by hand, and in fact is embedded in the synthetic division algorithm. Algorithms for solving systems of equations may likewise differ if calculators are available; we may even wish to teach new algorithms for multiplication and division with whole numbers, for instance.

A third criterion in the choice of an algorithm is its contribution to students' mathematical intuition. For example, a commonly used multiplication algorithm depends on the distributive property. The pupil is to think of $\begin{array}{r} 52 \\ \times\ 3 \end{array}$ as first 3 X 2 and then 3 X 50 (or 3 X 5, depending on the algorithm). In the commonly used multiplication algorithm for two two-digit numbers, 13 X 52 is written with partial products:

$$\begin{array}{r} 52 \\ \times\ 13 \\ \hline 156 \\ 52 \\ \hline 676 \end{array} \quad \text{or} \quad \begin{array}{r} 52 \\ \times\ 13 \\ \hline 156 \\ 520 \\ \hline 676 \end{array}$$

When such products are computed on even the simplest of calculators, distributivity is completely lost: 13 X 52 is computed as simply and quickly as 3 X 5. This may have significant advantages in illustrating the difference between the *operation* of multiplication and an *algorithm* for computing the product. However, the usual elementary school mathematics curriculum includes a considerable amount of practice on one or another multidigit multiplication algorithm, whether or not the distributive property on which the algorithm depends is ever explained. If this experience is eliminated by the pervasive use of calculators, will there be a loss in students' intuitive background in distributivity (which research indicates is already weak)?

Calculators invite the use of iterative algorithms. When should guessing and methods of successive approximations be taught? Such methods, providing for "built-in" checking, seem extremely promising as part of the problem-solving strand. Additional iterative algorithms should be adapted for school use.

New notation should be investigated. A standard, logical way to note calculator keystrokes is needed. The notation should be compatible with a useful notation for noncalculator mathematics. For instance, to compute $x = \frac{1}{2} \sin 2(a+b)$ a student might keystroke a [+] b [X] 2 [sin] [÷] 2 [=]. That has very little resemblance to the first notation. Consider the difference if we write keystrokes from right to left and use the reciprocal key and a parenthesis for $\frac{1}{2}$ (after all, the formula is "$\frac{1}{2}$ times something," not "something divided by 2"):

[=] [(] [1/x] 2 [)] [X] [sin] 2 [X] a [+] b.

This is much closer to the original algebraic notation, and even more similar to the computer language "A Programming Language" (APL) notion for the problem that might be used by older students, namely:

x ← (÷2) X 1O2 X a + b.

As new algorithms, curricular sequences incorporating these algorithms, and new notations are developed, research needs to be done on how well they accomplish various goals. Whether students are learning to use algorithmic processes, and not merely manipulate algorithms, is of key concern.

Recommendation 8. *Curriculum materials should be developed for kindergarten through twelfth grade to teach estimation, approximation, significant digits, order of magnitude calculations, and similar ideas.*

Discussion and Examples.--The importance of estimation skills has often been noted by mathematics educators, but printed curriculums have taken little note of this, nor have instructional practices reflected such concerns. Calculators give both new urgency to and new possibilities for estimation. For example, students should estimate answers so they can check the accuracy of their calculator work. This is of far more practical value than the often-heard suggestion that students use the calculator to check their work. The latter makes the machine the authority; the former asserts that the human being is in charge of a tool--a tool that needs to be checked. All materials for use with calculators should require that students ask themselves whether or not their answers are reasonable; in most cases that will involve an estimate.

All conference participants agreed that materials and pedagogical procedures must be developed to give students powerful ways of estimating and to teach students when an estimate, rather than a precise answer, is sufficient. But specific examples of how to do this were not elaborated. Clearly estimation must be emphasized at many places in the curriculum. Special planning and working groups might be commissioned to work out specific materials and teaching suggestions. Estimates with positive and negative powers of 10, use of "scientific notation," and order of magnitude estimation would surely become a part of such teaching.

Teaching about significant digits has been one of the things that many science teachers regard as important but mathematics courses generally ignore. The subject does not come up in the mathematical content of books and the problems typically found in mathematics books use such simplified (and unrealistic!) "data" that there is little reason for the student to wonder about which digits are significant. But the character of calculator displays, as well as the more realistic problems that use of calculators may engender, should alter this situation.

Calculators currently in use produce at least eight digits, and many users come to believe that all eight digits in the answer are important. Students must be taught specifically how to interpret answers in terms of situations--that is, in terms of the entered number and in terms of the application for the answer. For instance, consider the person who divides 241 (miles) by 7.8 (gallons of gasoline) and says that his car gets 30.897436 miles per gallon. Students need to understand that nothing except 30.9 can possibly be significant, and various errors possible in this situation make even that degree of significance unlikely. A person who thinks all eight digits matter and who cares about fuel economy might buy the wrong car. Using a calculator, a student can see what difference it makes if the amount of gas used was 7.83 or 7.79 gallons rather than 7.80. The student can also find that the difference between 30.897436 and 30.897434 miles per gallon will be about 0.03 gallons of gas in 15,000 miles. Thus the calculator, besides making an understanding of significant digits necessary, is an excellent tool for showing how changes in operands can change the answer.

Proposals need to be worked out for sequences of curriculum materials or pedagogical exercises dealing with significant digits. Such sequences could begin with intuitive understandings about why it is probably silly to keep too many significant figures or decimal places, and lead to intermediate work where the reasonable limits on answers are actually calculated. After this, some youngsters may acquire mature understanding of the theories and rules of thumb underlying computing practice with respect to significant figures. This last step might include a thorough study of the troublesome subject of round-off errors in large-scale computing.

A caution is necessary in relation to the finiteness of the calculator display. Materials may well be developed that take advantage of calculators to build students' intuitions about the structure and density of the real number system. But because every calculator has a finite display, using the calculator to divide 1 by 3, for instance, will always produce an incorrect answer. Any calculator-based curriculum must pay particular attention to this, so that students are aware of the mathematical difficulty. Research should be addressed to

questions related to it. For example, over a long period of time, will students' conception of number be confined to the Procrustean bed of 8, or 10, or 12 digits?

Recommendation 9. *Curriculum materials should be developed that teach problem-solving strategies more effectively and that build pupils' confidence in their ability to solve problems.*

Discussion and Examples.--Materials and pedagogical procedures are needed in which problem-solving strategies and techniques are systematically developed at all grade levels and in a wide variety of contexts, not all of which will be recognizable as "mathematical."

The calculator enables a student to focus on the strategies for solving problems, rather than the computation required to reach a solution. The use of problems with real data is now plausible at all levels. Because the primary goal of mathematics instruction is and has been to teach for use, the calculator provides a valuable tool for focusing attention on ends, rather than means.

Here again, conference participants agreed on the importance of this recommendation, but few specific examples of initiatives in response to it were offered because of time constraints. Giving encouragement to production and use of good problem material for all grade levels is surely one obvious response. Beyond that, some planning and working groups should be formed to suggest other initiatives responding to this recommendation.

Recommendation 10. *Curriculum materials should be developed and tested on new topics that are not now in the curriculum but that calculators may make feasible. The possibilities of adapting "advanced" topics to earlier levels should also be explored.*

Discussion and Examples.--As educators' viewpoints on mathematics change, new topics can be considered. By way of illustration, one working group put together the following example:

> *Suppose that "organization" (or structure) is considered as a basis of mathematics. A wide range of examples could support such a notion: reorganizing a set into linear order*

to count it; reorganizing two sets into one by forming their union (adding numbers); subdividing a set (factoring a number or an algebraic expression); such abstract concepts as factor group or fiber bundle. Mathematics thus becomes a study of different levels of organization and reorganization. With a viewpoint such as this, we can consider such new topics as logic, planning, and programing (not necessarily in the computer sense) as within the purview of the school curriculum; for example, in using a calendar or a schedule to plan one's time. But the logic of planning could also be considered specifically by mathematics teachers at various school levels. Working backward from an ending time to determine when subtasks must be started is a valuable skill that could become part of the mathematics program fairly early. Critical path analysis or other techniques of operations research could be taught specifically in relation to such applications as planning a trip or producing a yearbook. We feel sure that quite a variety of ideas about new topics could be elaborated and tested as schools come to terms with calculators.

In addition to introducing new topics, some topics might be reordered: that is, placed earlier or later in the curriculum than is currently the practice. For instance, decimal fractions might be taught 2 to 3 years earlier than they are now. As with all other curricular changes, research must be conducted to determine not only the feasibility of such changes but also their effect on understanding and achievement.

Care must be taken to consider and investigate the effect of change in subtle as well as obvious ways. For instances, suppose that, as decimal fractions are emphasized, work with common fractions decreases. But part of this work involves finding least common denominators and "reducing to lowest terms." A considerable amount of informal, intuitive work with primes, factors, multiples, and divisors could be eliminated--unless special steps are taken to ensure that these elementary number-theory topics are retained, since they may be important in later algebraic work. Using the calculator for practice on divisibility rules is one way of preserving such topics.

C. Curriculum Development for the Long-Range Future

As noted several times previously in this report, many of the curriculum development and research efforts for the short run will support longer range efforts. But short-run efforts are unlikely to be sufficient to ensure thoughtfully planned and substantially revised

responses to the availability of hand-held calculators (and later, hand-held minicomputers). Given the unfamiliar territory that must be explored here and the development time required in new curriculum initiatives, work on new initiatives must begin *now* if they are to be ready for use on a larger scale when needed some years from now.

Recommendation 11. *Some full-scale alternatives to the K-6 elementary school mathematics program should be developed that use calculators wherever appropriate (but not exclusively) and that considerably broaden the range of mathematical ideas absorbed during the elementary school program.*

Discussion and Examples.--In appendix F of the report by Suydam (1976), Usiskin and Bell discuss what is included in current mathematics curriculums used in a large proportion of elementary schools. The present report merely suggests a few alternatives to illustrate the range of full-scale development efforts that should receive support. Each would integrate the use of calculators fully.

Careful thought and the extensive use of research findings are essential to ensure that new curriculums will be appropriate and viable for the variety of schools, teachers, and students in this country. Any proposal for the development of such a curriculum should detail the procedures to be followed, although the potential for exploring unexpected avenues should also be open. Teachers and other school personnel should be involved in all stages as much as possible, from planning and design through development, testing, and research. While creativity is to be encouraged, previous research and development of curricular components, especially those incorporating the use of calculators, should be included where appropriate. (Each project does not have to reinvent the wheel!)

Among the alternatives that might be considered are the following:

1. Algorithmic approach: This would include a thorough study of known algorithms and procedures (not only for arithmetic calculations) and would involve creativity in developing new ones. Understanding algorithmic processes would be a key goal. This relates in obvious ways to recommendation 7.

2. Integrated project approach: This approach would collect topics that cut across several academic and nonacademic areas. Science, industrial arts, social sciences, health science, and physical education could be involved (although not all at one time). The instruction might be conducted by a team of teachers, each serving as a resource. Materials mined by response to recommendation 1 from innovative science and other curriculum projects of the 1960's and 1970's (e.g., USMES, ESS, MINNEMAST) could provide important resources to get such an approach under way.
3. Games-simulation approach: Here programmable calculators or computers would become major tools. Through the use of graphic displays and possibly servomechanisms such as those suggested by S. Papert,[1] preprogramed instructional sequences, and so on, a totally new set of methodologies could come into play.
4. Approach based on some specific psychological or learning theory: While all approaches would incorporate or draw from some theory or theories, this approach would attempt to put a specific theory into practice. Within the bounds set by the need to present sound mathematics, the theory would determine such factors as the sequencing and the methods of presenting and reinforcing learning.

Well formulated proposals for other novel approaches should be encouraged.

Recommendation 12. *Some new courses for secondary school students should be developed, including consumer-industrial and data-oriented statistics courses, as well as alternatives for junior high school work and for those high school students who do not elect standard "college preparatory" mathematics courses.*

Discussion and Examples.--Much of the discussion of recommendation 11 about developing new courses applies here. Some of the approaches suggested there could be used on the secondary school level.

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[1] *See, for example, Molnar 1976, Papert 1973, Goldstein, et al., 1976.*

For example, the study of algorithms could be extended to at least algebra and trigonometry, and the project approach or games-simulation approach can be used above the elementary school level.

As noted in the introduction, modifications in the mainstream college-preparatory high school curriculum will probably be accomplished by ongoing efforts by textbook authors and publishers. Following are a few alternative course suggestions; more should be encouraged:

1. Consumer-industrial approach: This would use local business and industry for data sources. It could begin in the middle-school years, or it could be an alternate to the present mainstream of high school courses. It could involve work-study programs.
2. Data-oriented statistics approach: A statistics course with a heavy emphasis on sensible handling of numerical data is particularly needed. The NACOME report argues persuasively for development of such a course (Hill et al 1975).
3. College-preparatory statistics approach: A junior- or senior-level statistics and probability course for "college capable" students has been advocated at least since the Commission on Mathematics report. With programmable calculators it now seems possible to devise such a course that could focus on realistic problems and the underlying meanings of various statistical tests and routines in ways not possible before. Such a course could well become an alternate "advanced placement" option that could allow students with certain educational and career interests to take more sophisticated courses early in their college training than most are now able to take (CEEB 1959).

Well formulated proposals for other novel approaches and courses should be encouraged.

D. Research and Evaluation

As noted in the introduction, the conferees concluded that research and evaluation must interact with curriculum development at every point. The participants identified the following additional concerns.

Recommendation 13. *Sampling and research methodology should be appropriate to the type and scale of development. Methods can range from clinical trials to full-scale randomized intervention experiments, and from research focused on special subsets of the school population (e.g., remedial, handicapped, gifted) to that focused on a more general population.*

Discussion and Examples.--Such subsets include those varying in achievement (low achieving or remedial, gifted, and "regular" students) and those with special conditions (physically handicapped, mentally retarded, learning disabled, and emotionally disturbed students), at all grade or age levels. Perhaps these students have the same mathematical needs and should use the same calculator curriculum materials as other students. Perhaps the same materials can be used, but the time of introduction, the strategy of use, or other factors need to be varied.

Recommendation 14. *Current psychological, behavioral, and learning theory models should be surveyed to identify what they might contribute to the curriculum development process.*

Discussion and Examples.--What findings from research on developmental levels, motivation theory, reinforcement theory, information-processing theory, and so forth, are applicable? To answer such questions, psychologists and educators need to conduct critical analyses of research and of curriculums. Experimental studies, conducted in the course of curriculum development, should address questions related to various theories. Here are a few examples of such questions:

1. How well does an information-processing model describe the way a child processes an algorithm with a calculator?
2. How do children at various developmental levels (e.g., those identified by Piaget) form understandings about decimals when using a calculator?
3. Is the process of using the calculator to manipulate symbols similar to that of using concrete manipulative materials?
4. What is the effect of the calculator on a child's early ideas of number?

5. How much reinforcement of mathematical skill and algorithmic learning is necessary when calculators are used?

Recommendation 15. *There should be development and research focused on new approaches to achievement testing. Testing policy should be analyzed and appropriately changed in response to these results.*

Discussion and Examples.--Many persons have expressed concern about school testing programs in general, and more specifically about the effect of the use of calculators in relation to mathematics tests. Tests should be analyzed to ensure their appropriateness to the specific curriculums in use, including those in which calculators are used. Standardized test items must be developed to match changing curricular emphases resulting from the use of calculators, such as estimation skills, problem-solving strategies, and new topics. If calculators are used on standardized tests, the types of answer choices will need to be reconsidered, as will the fact that students have various types of calculators. Norms will need to be determined both for populations using calculators for instruction and those using calculators on the test.

Whether using calculators is allowed on classroom tests is and probably will remain a local school or teacher option. Two key factors to be considered in making this decision are (1) whether all students have calculators and (2) what objectives are being tested. Teachers, curriculum developers, and researchers must continue to select and develop tests so that the content is appropriate to the mathematics being taught. Studies might be designed to investigate the effects of whether or not calculators are used on tests.

Recommendation 16. *New approaches to assessing attitudes and other noncognitive outcomes of school mathematics instruction should be studied, and curriculums using calculators should be subject to such assessment.*

Discussion and Examples.--As students use calculators in varying ways with various curricular materials, many facets of their affective reactions must be measured. To do this, a diversity of techniques

should be used, including interviews, written comments, observations, anecdotal records, and scales. Many of these will need to be constructed explicitly to assess affective concerns. Many questions are possible, of course; here are some examples:

1. To what extent is the use of calculators in given ways motivational? For which students, if any, is the use of calculators particularly effective in terms of affective concerns?
2. How consistent are motivational levels over time?
3. To what extent do various topics or procedures have differing effects on motivational level and which practices are particularly effective or ineffective?
4. How do attitudes toward mathematics vary over time as students use calculators?
5. To what extent does use of calculators reduce students' fear of mathematics when it exists?
6. How does use of calculators affect students' feelings of control of their environment, anxiety, alienation, and other affective factors?

Recommendation 17. *Every effort should be made to assess any changes in the amount of time spent in mathematics instruction and the distribution of that time as calculators become an integral part of the school experience.*

Discussion and Examples.--It has been hypothesized that less time will be needed to teach the mathematical ideas in current curriculums as calculators are used. Therefore a study might be designed to determine how the time for learning mathematics varies when calculators are used. If the variance is considerable, then curriculum developers should consider how the available time could best be allocated to improve the learning of mathematics. Attention must be given to the implications for reordering and restructuring curriculums. Response to this recommendation might well be incorporated in the surveys and studies that respond to recommendation 2.

E. Teacher Education

While only one recommendation pertains specifically to teacher education, this recommendation interacts with most of the other recommendations. Outcomes of those recommendations cannot succeed unless teachers are knowledgeable about, as well as participate in, curricular change.

Recommendation 18. *Training and retraining of teachers to help them respond to calculators and calculator-influenced curriculum materials must be made equal in importance to the development of initiatives aimed at school-age children.*

Discussion and Examples.--In the curriculum revision process of the past few decades, the overwhelming importance of teacher education was rarely fully recognized. The huge population of elementary school teachers was, in particular, virtually ignored. At the secondary school level, two-thirds of the teachers participated in funded institutes, and at least some teachers were actively involved in curriculum development projects. But elementary school teachers rarely had the opportunity to participate in the curriculum development process; they rarely were the funded participants in in-service institutes. Instead, because of their numbers, they were frequently given mathematics training second-hand.

Consequently, as is noted in the NACOME report, there is little evidence of success with curriculum reform at the elementary level. Furthermore, experience gleaned from major educational reform and change efforts provides clear evidence that teachers and principals must be knowledgeable about and must support such efforts if the efforts are to succeed. Therefore, every effort should be made not only to inform but also to involve teachers in the process of developing and testing curricular and instructional ideas.

The highest priority is to provide resources for elementary teachers, for they are the ones who must cope with the calculator as a tool that has the potential of changing the computational focus of the elementary school curriculum. One participant suggested a series of 1-week institutes, designed to reach *every* elementary teacher, with longer institutes available for those who can take advantage of them. Another way in which teachers can be reached is through teacher centers. In

such centers, resources are collected and teachers share ideas basic to making the teaching process more effective.

Such intensive work on calculators must also be included in the pre-service teacher education program. Educators cannot afford to avoid or delay the inclusion of essential new topics in the preparatory program. Colleges must not graduate beginning teachers who have insufficient background to cope with immediate pedagogical problems. As at the in-service level, essential components of this instruction include changes in curricular topics and emphases, effective instructional procedures for using calculators, hands-on experiences with many types of calculators, and understanding the reasons for change. Teachers need to be aware of the need to teach the child to live in a changing technological world; they must be fully aware of how to incorporate successful uses of calculators, as well as other technological tools, in the curriculum.

In-service work as well as pre-service work must be planned for secondary teachers, also. But secondary teachers generally have a more extensive background in mathematics than do elementary teachers, and thus many have already begun to incorporate calculator applications in their teaching. Moreover, the secondary mathematics curriculum is not as intrinsically affected by the calculator. Thus teachers at this level could be reached in less direct ways (e.g., through printed materials), in order to concentrate resources at the level of most pressing need.

F. Dissemination

This category differs from the preceding ones in its focus on facilitating change by attending to easy and timely *exchange* of information and materials about use of calculators rather than the actual *production* of information and materials.

Recommendation 19 concerns the establishment of an information collection and dissemination center as an essential first step. Recommendations 20, 21, and 22 call for developing ways to facilitate the exchange of information among and between educators, parents and public, textbook publishers, calculator manufacturers, and other interested persons. All groups should be aware of (1) the rapid technological changes taking place; (2) efforts to produce curricular components and

curriculums appropriate for various instructional uses of calculators; and (3) results of research conducted to ascertain effects of various school uses of calculators.

Recommendation 19. *A central information collection and dissemination center should be established.*

Discussion and Examples.--The information collection and dissemination process is important in furthering appropriate development and use of calculator materials by coordinating research and development efforts, avoiding needless duplication, and providing a source of knowledge and assistance. At present some aspects of this function are informally shared by the National Council of Teachers of Mathematics and the ERIC Center for Science, Mathematics, and Environmental Education. However, national attention has not been sufficiently directed to either as a place to which to send information on research, curricular, and technological activities. Also, at present neither has the resources that would be required for the job that needs to be done.

The center would do at least four things, as follows:

1. Develop an information data base. Information on calculator activities in such places as local school systems, State agencies, universities, and industry will be routinely routed to the center, and it will actively seek additional information.
2. Develop an easy way to gain access to the information. The center should provide summaries of "what's going on where," as well as copies of materials or specific sources for additional information.
3. Monitor technological developments.
4. Develop press releases, articles, and perhaps a newsletter to keep educators and the public informed.

One of the conference working groups urged that, as one source of input to the information center, manufacturers of calculators sponsor research and share research findings about the effect of using calculators in schools, as well as share pertinent technological information. It also urged that publishers study the effect of calculator-oriented curriculum materials and share information on their findings.

"In a time of rapid and important change, information should not be withheld for merely competitive advantage, or to gloss over possible unfavorable effects. That is, relatively complete sharing of information is essential," the group stated.

Recommendation 20. *Ways should be explored to inform the public about how schools are adapting to calculators.*

Discussion and Examples.--Most educators are diffident about explicitly using public relations or publicity mechanisms in dealing with the public. Such diffidence is on the whole commendable. But because of the prospects for substantial change in teaching arithmetic in the next few years, it is probably prudent to try to keep the public informed as these changes take place. The following suggestions were made as possible ways to do this:

1. Summaries of information collected by the Calculator Information Center could be disseminated through releases to print and broadcast media, and through public forums, including but not limited to parent organizations.

2. A syndicated newspaper column could be developed, containing puzzles, recreations, and typical consumer applications using calculators. The publication of paperbound books on calculator games and articles in popular magazines about "how to get more out of your calculator" indicate that there would be readers for such a column, which might also include references to school developments.

3. Information bulletins can be sent home with children, to keep parents regularly informed.

4. Forums could be arranged between local parent groups and informed members of the school community to discuss issues and plans for program revision.

5. A speakers' and consultants' bureau might be established to provide experts who can participate in such forums.

6. With or without calculators, public misinformation about mathematics is a barrier to improvements in mathematics education generally and to accommodation of the school program to calculators in particular. One of the working groups of the conference urged a

determined effort to meet this problem head on, in the following suggestion:

> *We suggest that a television series on the mathematical sciences be developed. What "Nova" and "The Ascent of Man" have done for the natural, physical, and anthropological sciences, and what "America" and "The Adams Chronicles" have done for U.S. history, needs to be done for the mathematical sciences. We propose a general series, for public viewing, emphasizing the breadth, power. beauty, utility, and pervasiveness of mathematics in our lives. This would have the potential of building positive attitudes, inspiring further study of mathematics, and emphasizing the fact that computation is just a small part of a fascinating body of useful knowledge. (Thus, the calculator would be put into perspective.) In addition to use on television, tapes of the series could be made available for school use along with teaching guides for related learning activities. This is not a task to be undertaken lightly: people of the caliber of Jacob Bronowski and Alistair Cooke are few and far between, and this series must be of the highest quality.*

Recommendation 21. *New means should be explored to rapidly communicate results of experiments with calculators and proposals for their use to the teaching profession, especially to elementary school teachers.*

Discussion and Examples.--Although many teachers are involved in teaching mathematics at the elementary and secondary school levels, the number of national journals that deal with mathematics education is small. Middle school and high school teachers are fairly well served by several high quality journals, even though there is typically considerable delay between submission and publication of articles and even though the space restrictions are severe. But for the vast majority of elementary school teachers--nearly all of whom teach mathematics--there is no regular, reliable source of timely information on mathematics education. (*The Arithmetic Teacher* is a fine journal, but only a tiny percentage of the roughly 1.2 million elementary school teachers subscribe to it.) There are periodicals that reach larger numbers of elementary school teachers, but for the most part, coverage of mathematics education concerns in these is scant and poorly informed.

Conferees saw this as an important matter but had few suggestions for dealing with it. Perhaps this is an area where a small working group could be profitably convened to make suggestions and recommendations. Here are some suggestions that were made at the conference:

1. The Calculator Information Center should publish a regular newsletter abstracting research and development results in ways helpful to teachers. The Center should ensure the newsletter's wide distribution, especially among elementary school teachers.

2. The National Council of Teachers of Mathematics and other publishers of mathematics education journals might be given grants to periodically expand the number of pages in given issues to accommodate special and timely supplements about calculator-related developments.

3. Reports of research and development efforts appear in professional journals and other sources that are often relatively inaccessible to most teachers. Agencies such as the Calculator Information Center, NCTM, or NIE should periodically make collections of the best of such items available for wide distribution.

4. Ways should be found to expand and improve the coverage of mathematics education in general-purpose education publications read by large numbers of elementary school teachers. Perhaps subsidies to support special calculator-related supplements would also be appropriate here.

Recommendation 22. *Means should be found to ensure access to critical evaluations of calculator-oriented materials proposed for school use, and to provide information and guidelines about the calculators themselves.*

Discussion and Examples.--Materials to be evaluated would include all calculator curricular sequences, published supplementary materials, and text materials. To some extent, such materials are evaluated by reviewers in NCTM journals, for example. But the time lag can be consequential, and not all materials are submitted for such reviews.

Evaluation and critical review are not necessarily among the primary functions of the Calculator Information Center, though such efforts would surely interact with both the collection and dissemination

functions of that center. Therefore, other means for providing users with information on the scope and quality of calculator-related materials should be devised.

Given the diversity of calculators available, the highly competitive nature of the calculator industry, the experience of some school districts and parents who were sold low-quality or energy-expensive machines, and the sale of unevaluated curricular materials, the development of guidelines seems essential. State supervisors and others in school-related positions should attack this problem as soon as possible. Criteria might include readability of instructions, guarantees of minimum production continuation (e.g., for 3 years), energy-use standards, and quality curricular materials. Suggestions might also be developed for the design of calculators specifically for educational purposes; these should take into account guidelines already prepared by some State groups. At least over the short run, it may even be advisable to provide for some sort of consumer information capability that can provide unbiased and technically sophisticated information on calculator and similar microelectronic products available to the school market.

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SUMMARIES OF PRESENTATIONS OF RESOURCE PERSONS

Resource Persons

Bryan Bunch	American Book Company
Jay V. Hemming	National Semiconductor Corporation
Ruth Hoffman	University of Denver
Andrew Molnar	National Science Foundation
Ralph Oliva	Texas Instruments Incorporated
Marshall Smith	National Institute of Education
John Stubbs	Maryland Chapter of the NAACP
J. Fred Weaver	University of Wisconsin at Madison

Bryan Bunch, Senior Vice President and Editor-in-Chief of the American Book Company

Mr. Bunch, who was suggested by the Association of American Publishers, began his presentation by saying that publishers are not very experienced in dealing with the type of innovation that includes handheld calculators. Calculators are not a *body of content* (such as the metric system, for example), which publishers are accustomed to handling. Nor are they a *method* (such as individualization), nor a *medium* (such as filmstrips).

Publishers have three choices: (1) ignore calculators (which is very unlikely); (2) "exploit" them, before adequate research is done; or (3) use them as a basis for creative, innovative mathematics instruction. A period of "exploitation" will be inevitable, according to Mr. Bunch, simply because publishing companies, which often require 5 years to go from initial planning to the final production of a textbook series, will make mistakes in trying to be among the first to publish in this field.

Mr. Bunch foresaw several curricular changes in basic programs, including earlier and more extensive work with decimals, more emphasis on estimation, and perhaps emphasis on iterative procedures in high school. He also said that publishers would be unlikely to risk an entirely new, calculator-dependent, mathematics series for some time, because it might not be acceptable to schools.

Among the things that Mr. Bunch would like to see in the near future are the development of a body of clever ideas for using calculators in K-12 classrooms, and research on how those ideas work.

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Jay V. Hemming, Educational Marketing Manager, Consumer Products Division, National Semiconductor Corporation

Mr. Hemming began by outlining the "calculator revolution" in terms of annual sales figures. Industry sources forecast annual calculator sales of $100 million by 1980. Calculators have been appearing in classrooms with or without educators' approval, and will continue to do so.

This means that educators are in the unaccustomed position of having to respond very rapidly to questions concerning how and when the

calculator should be used in teaching mathematics, Mr. Hemming said. Further, these questions will constantly change as new devices (including microprocessors, hand-held computers, and home computers) become widespread. Surely educators will no longer have the luxury of spending 5 or 10 years to investigate hypotheses concerning these technical breakthroughs.

According to Mr. Hemming, there is little concern among manufacturers about how calculators are used in classrooms. This stems principally from the manufacturers' lack of understanding and awareness of the problems and procedures of the educational system.

Mr. Hemming noted that at present there is no industrywide group among the calculator manufacturers similar to the publishers' Association of American Publishers. He concluded that one of the most urgent needs is for cooperation and coordination among educators, publishers, and calculator manufacturers.

Mr. Hemming strongly recommended the establishment of the National Clearing House for research on the classroom use of calculators so that wasteful duplication of effort will not slow progress.

* * *

Ruth Hoffman, Professor of Mathematics and Director of the Mathematics Laboratory, University of Denver, and
Ralph Oliva, TI Learning Center, Texas Instruments Incorporated

Ms. Hoffman is one of the authors of a program being developed in conjunction with Texas Instruments Incorporated. She and Mr. Oliva gave an overview of the Texas Instruments programs. These include a modularized series of supplementary activities for use in kindergarten and first grade, an introductory activity unit for grades 4 through 6, a supplementary program for 7th and 8th grade fundamental mathematics curriculums, and an enrichment package for introductory algebra courses.

Ms. Hoffman spent a short time describing the traditional mathematics curriculum for grades 7 through 12, indicating for each grade level how much time might be saved if calculators were used fully. According to her estimates a very substantial portion of the present curriculum in those grades would be affected if calculators were used extensively.

Mr. Oliva described several technological advances that he saw in the not-too-distant future, including (1) wider use of the Algebraic Operating System, (2) cheaper thermal printing devices, (3) the magnetic bubble memory, (4) an inexpensive programmable calculator with 30 or 40 program steps, and (5) the pervasive use of logic and memory in all kinds of familiar devices like stoves. As a result, Mr. Oliva asserted, programing will become even more important than it is now.

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Andrew Molnar, Science Education Directorate, National Science Foundation

Mr. Molnar's presentation was concerned more with the broader aspects of computing technology than with calculators. He described his experiences with systems like PLATO and TICCIT, and with regional computing networks like the one based at Dartmouth College.

Mr. Molnar urged the conference participants to think beyond the hand-held calculator--to consider, for example, the possibilities that would be opened up if the hand calculator were used as a plug-in device to a small computer, or plugged into the home television set. The problem is not how to use the hand calculator, but how to use computer-based algorithmic problem solving. The entire mathematics curriculum should be reviewed to determine where algorithms such as square root should be automated and other procedures assigned to algorithmic problem solving using hand-held calculators.

Mr. Molnar predicted that within 5 years there will be machines that combine the video disc with the microprocessor. Soon one will be able to obtain in a device costing less than $1,000 and weighing less than 25 pounds the power currently contained in a medium-sized computer. It is not inconceivable that we will have dozens of hand calculators in schools, some for mathematics, some for English, poetry, or any other topic.

Mr. Molnar also described some of the more recent work done by Seymour Papert at MIT, which has gone far beyond the original "turtle."

* * *

Marshall Smith, Associate Director of the Office of Planning, Budget, and Program Analysis, National Institute of Education

Participants at the conference were interested in methods of determining the effects that calculators might have on precollege students, particularly when calculators are used in relatively unstructured ways in the classroom. In response to these interests, Mr. Smith began his remarks by describing some of his experiences in reanalyzing data, including data from the Coleman study and from the Westinghouse Head Start study. Mr. Smith pointed out that stronger results could be obtained from studies in which some experimental intervention occurred, as in the Head Start planned variation study.

Mr. Smith described two steps that one could take in carrying out a study of the effects of calculators. The first is to gather good descriptive data as a guide to the second step, which is the intervention itself. An example of one possible experimental intervention would be to give selected superintendents money to buy calculators. The goal would be to simulate under controlled conditions what might occur naturally.

Mr. Smith said that in this sort of experiment there is no need to work with a nationally representative sample. It is more important to be completely confident that the experimental intervention is actually being carried out. He stressed also the importance of the kinds of tests used to measure changes in students' performances.

* * *

John Stubbs, representative of the Maryland chapter of the National Association for the Advancement of Colored People (NAACP)

Mr. Stubbs said that the Maryland chapter of the NAACP was instrumental in introducing a bill into the Maryland legislature that would ban the use of hand-held calculators in schools within the State. He said that this was a result of his organization's deep concern for, and commitment to, quality education.

The Maryland NAACP feels that all too often, when some kind of educational innovation proves to be harmful, it is the black students who get the worst of it. The central question with regard to calculators is whether they will help to develop the step-by-step logic that

is necessary in later life. Mr. Stubbs said that the opinion of his group is that, to the contrary, calculators serve only as a crutch, reducing students' ability to understand basic mathematics, partly by reducing the amount of drill and practice that children get.

Members of the Maryland chapter of the NAACP are concerned not so much with the short-term effects as with the effects of perhaps 10 years of calculator usage. While granting that their position is not based on "hard data," Mr. Stubbs said that the proponents of calculators have no evidence to support their view, either. Until such evidence is available, the Maryland NAACP prefers its present approach, preventing what could be irreparable harm.

If it could be demonstrated that hand-held calculators have beneficial effects in mathematics instruction, then the group would definitely support their use, according to Mr. Stubbs.

* * *

J. Fred Weaver, Professor of Curriculum and Instruction, University of Wisconsin at Madison

Mr. Weaver's main points centered around the elementary grades. He argued that the least profitable and effective course to take would be to accept the usual curriculum as it is and find ways that calculators can be used within that curriculum. In current practice, so-called "word problems" and even the study of properties of number systems are tied to computational skill. Now, for the first time, the curriculum need not be constrained by the developments of those skills, and Mr. Weaver urged that we take advantage of the situation to pursue some bolder innovations.

He claimed that we need to look at two *different* things: (a) mathematical ideas, and (b) paper-and-pencil computation. As an example, he described a "guess and test" approach in second and third grades, where children are confronted with situations like the following: $\square \xrightarrow{+394} 871$. Here the emphasis is on the *unary* operation +394 rather than on the binary operation of addition. This idea can be explored prior to mastery of a subtraction algorithm.

Part of the ensuing discussion focused on the issue of standardization of machines (logic, keyboards, and so on). Mr. Weaver felt that

lack of standardization is currently a handicap in elementary teacher education, but presents no problems for secondary teachers.

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